# THE SCHILLING SCHOOL FOR GIFTED CHILDREN

2008-2009 Academic Year

## **CALCULUS / POST CALCULUS**

Dr. Douglas G. Frank, Instructor

CLASS TIME: Monday & Wednesday Mornings

### **TEXT: Calculus and Analytic Geometry (9th Edition)**

by George B. Thomas and Ross L. Finney



**OUTLINE AND OBJECTIVES:** After several introductory classes to review material and establish ability, philosophy, and intuition, students will systematically progress through Finney at differentiated paces. Much of what we learn will be immediately applied in our *Applied Science and Mathematics* course, and the classes will often be integrated.

The students enrolled in this class appear to be divided into two primary groups; some with experience in calculus, and some with very little. Levels of experience will be established, and the class will progress in a differentiated fashion.

### Some initial material will include:

- Mathematics vs. Arithmetic
- Mathematics as Language
- Thinking in Calculus
- Applications of the Calculus
- Simulating Calculus with spreadsheets, Practical Computing
- Exploration of differential and periodic phenomena

Students will be required to keep notes in class, and will turn in homework on three-hole punched paper each Monday for credit. Homework will be assigned from the text, with additional challenge problems that I will assign <evil grin>. Exams will be given approximately every three weeks, with pop-quizzes. Grades will be based equally on exams and homework.

#### Important: Students MUST bring to each class:

- Three-ring binder and paper
- Sharpened pencils or mechanical pencil and erasers
- Calculator (graphing calculators discouraged. A \$10 science/engineering calculator will be fine.)
- Textbook

Laptops are optional, but are welcome and encouraged.

The table of contents for the text follows.

		To the Instructor viii To the Student xvii	
P	Preliminaries	<ol> <li>Real Numbers and the Real Line 1</li> <li>Coordinates, Lines, and Increments 8</li> <li>Functions 17</li> <li>Shifting Graphs 27</li> <li>Trigonometric Functions 35</li> <li>QUESTIONS TO GUIDE YOUR REVIEW 47 PRACTICE EXERCISES 48</li> <li>ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 49</li> </ol>	
1	imits and Continuity	<ul> <li>1.1 Rates of Change and Limits 51</li> <li>1.2 Rules for Finding Limits 61</li> <li>1.3 Target Values and Formal Definitions of Limits 66</li> <li>1.4 Extensions of the Limit Concept 78</li> <li>1.5 Continuity 87</li> <li>1.6 Tangent Lines 97</li> <li>QUESTIONS TO GUIDE YOUR REVIEW 103 PRACTICE EXERCISES 10 ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 105</li> </ul>	04
2 (	Derivatives	<ul> <li>2.1 The Derivative of a Function 109</li> <li>2.2 Differentiation Rules 121</li> <li>2.3 Rates of Change 131</li> <li>2.4 Derivatives of Trigonometric Functions 143</li> <li>2.5 The Chain Rule 154</li> <li>2.6 Implicit Differentiation and Rational Exponents 164</li> <li>2.7 Related Rates of Change 172 <ul> <li>OUESTIONS TO GUIDE YOUR REVIEW 180</li> <li>PRACTICE EXERCISES 1</li> </ul> </li> </ul>	181
3	Applications of Derivatives	ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 185 3.1 Extreme Values of Functions 189 3.2 The Mean Value Theorem 196 3.3 The First Derivative Test for Local Extreme Values 205	

iii

		3.4 3.5 3.6 3.7 3.8	Graphing with y' and y'' 209 Limits as $x \rightarrow \pm \infty$ , Asymptotes, and Dominant Terms 220 Optimization 233 Linearization and Differentials 248 Newton's Method 260	
_			QUESTIONS TO GUIDE YOUR REVIEW 268 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 272	269
4	Integration	4.1 4.2 4.3 4.4 4.5 4.6 4.7 4.8 4.9	Indefinite Integrals 275 Differential Equations, Initial Value Problems, and Mathematical Modeling Integration by Substitution—Running the Chain Rule Backward 290 Estimating with Finite Sums 298 Riemann Sums and Definite Integrals 309 Properties, Area, and the Mean Value Theorem 323 The Fundamental Theorem 332 Substitution in Definite Integrals 342 Numerical Integration 346 QUESTIONS TO GUIDE YOUR REVIEW 356 PRACTICE EXERCISES	282
5	Applications of Integrals	5.1 5.2 5.3 5.4 5.5 5.6 5.7 5.8 5.9 5.10	Areas Between Curves 365 Finding Volumes by Slicing 374 Volumes of Solids of Revolution—Disks and Washers 379 Cylindrical Shells 387 Lengths of Plane Curves 393 Areas of Surfaces of Revolution 400 Moments and Centers of Mass 407 Work 418 Fluid Pressures and Forces 427 The Basic Pattern and Other Modeling Applications 434 QUESTIONS TO GUIDE YOUR REVIEW 444 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 447	444
6	Transcendental Functions	6.1 6.2 6.3 6.4 6.5 6.6 6.7 6.8 6.9 6.10 6.11 6.12	Inverse Functions and Their Derivatives 449 Natural Logarithms 458 The Exponential Function 467 <i>a</i> ' and log <sub>a</sub> x 474 Growth and Decay 482 L'Hôpital's Rule 491 Relative Rates of Growth 498 Inverse Trigonometric Functions 504 Derivatives of Inverse Trigonometric Functions; Integrals 513 Hyperbolic Functions 520 First Order Differential Equations 529 Euler's Numerical Method; Slope Fields 541 QUESTIONS TO GUIDE YOUR REVIEW 547 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 551	548
7	Techniques of Integration	7.1 7.2	Basic Integration Formulas 555 Integration by Parts 562	

		7.3 7.4 7.5 7.6	Partial Fractions 569 Trigonometric Substitutions 578 Integral Tables and CAS 583 Improper Integrals 594 QUESTIONS TO GUIDE YOUR REVIEW 606 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 609	606
8	Infinite Series	8.1 8.2 8.3 8.4 8.5 8.6 8.7 8.8 8.9 8.10 8.11	Limits of Sequences of Numbers 613 Theorems for Calculating Limits of Sequences 622 Infinite Series 630 The Integral Test for Series of Nonnegative Terms 640 Comparison Tests for Series of Nonnegative Terms 644 The Ratio and Root Tests for Series of Nonnegative Terms 649 Alternating Series, Absolute and Conditional Convergence 655 Power Series 663 Taylor and Maclaurin Series 672 Convergence of Taylor Series; Error Estimates 678 Applications of Power Series 688 QUESTIONS TO GUIDE YOUR REVIEW 699 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 703	700
9	Conic Sections, Parametrized Curves, and Polar Coordinates	9.1 9.2 9.3 9.4 9.5 9.6 9.7 9.8 9.9	Conic Sections and Quadratic Equations 709 Classifying Conic Sections by Eccentricity 723 Quadratic Equations and Rotations 728 Parametrizations of Plane Curves 734 Calculus with Parametrized Curves 744 Polar Coordinates 751 Graphing in Polar Coordinates 756 Polar Equations for Conic Sections 764 Integration in Polar Coordinates 770 QUESTIONS TO GUIDE YOUR REVIEW 777 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 783	778
10	Vectors and Analytic Geometry in Space	10.1 10.2 10.3 10.4 10.5 10.6 10.7	Vectors in the Plane 787 Cartesian (Rectangular) Coordinates and Vectors in Space 795 Dot Products 806 Cross Products 815 Lines and Planes in Space 822 Cylinders and Quadric Surfaces 829 Cylindrical and Spherical Coordinates 841 QUESTIONS TO GUIDE YOUR REVIEW 847 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 851	848
11	Vector-Valued Functions and Motion in Space	11.1 11.2 11.3 11.4 11.5	Vector-Valued Functions and Space Curves 855 Modeling Projectile Motion 868 Arc Length and the Unit Tangent Vector T 876 Curvature, Torsion, and the TNB Frame 881 Planetary Motion and Satellites 893 QUESTIONS TO GUIDE YOUR REVIEW 902 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 905	902

#### vi Contents

-

-				
12	Multivariable Functions and Partial Derivatives	12.1 12.2 12.3 12.4 12.5 12.6 12.7 12.8 12.9 12.10	Functions of Several Variables 909 Limits and Continuity 917 Partial Derivatives 924 Differentiability, Linearization, and Differentials 933 The Chain Rule 944 Partial Derivatives with Constrained Variables 952 Directional Derivatives, Gradient Vectors, and Tangent Planes 957 Extreme Values and Saddle Points 970 Lagrange Multipliers 980 Taylor's Formula 989 QUESTIONS TO GUIDE YOUR REVIEW 993 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 998	994
13	Multiple Integrals	13.1 13.2 13.3 13.4 13.5 13.6 13.7	Double Integrals 1001 Areas, Moments, and Centers of Mass 1012 Double Integrals in Polar Form 1020 Triple Integrals in Rectangular Coordinates 1026 Masses and Moments in Three Dimensions 1034 Triple Integrals in Cylindrical and Spherical Coordinates 1039 Substitutions in Multiple Integrals 1048	
_			QUESTIONS TO GUIDE YOUR REVIEW 1055 PRACTICE EXERCISES ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 1058	1056
14	Integration in Vector Fields	14.1 14.2 14.3 14.4 14.5 14.6 14.7 14.8	Line Integrals 1061 Vector Fields, Work, Circulation, and Flux 1067 Path Independence, Potential Functions, and Conservative Fields 1076 Green's Theorem in the Plane 1084 Surface Area and Surface Integrals 1096 Parametrized Surfaces 1106 Stokes's Theorem 1114 The Divergence Theorem and a Unified Theory 1123 QUESTIONS TO GUIDE YOUR REVIEW 1134 PRACTICE EXERCISES 1134 ADDITIONAL EXERCISES—THEORY, EXAMPLES, APPLICATIONS 1137	
	Appendices	A.1 A.2 A.3 A.4 A.5 A.6 A.7 A.8 A.9	Mathematical Induction A-1 Proofs of Limit Theorems in Section 1.2 A-4 Complex Numbers A-7 Simpson's One-Third Rule A-17 Cauchy's Mean Value Theorem and the Stronger Form of l'Hôpital's Rule Limits That Arise Frequently A-20 The Distributive Law for Vector Cross Products A-21 Determinants and Cramer's Rule A-22 Euler's Theorem and the Increment Theorem A-29	A-18
		Index	1-1	

A Brief Table of Integrals T-1