Units

Most everything we observe and measure can be described in terms of number, time, distance, and mass. Although all quantities are related via units of energy, in practice we distinguish between fundamental units and derived units, where derived units are composed of the fundamental units of time (seconds, s), distance (meters, m), and mass (kilograms, kg). Some examples:

Observed Quantity	Symbol *(depends upon context)*	Units	What Are We Really Measuring?
Number	n		How many are there?
Time	t	s	The time interval between two events
Rate	R	/s	How many per second
Length	L	m	The distance between two locations (points)
Position		(m, m, m,…)	You need to specify one length for every dimension in your system in order to specify a position relative to the orgin of that system. Angular coordinates can replace lengths.
Mass	m	kg	How much 'stuff' there is
Speed	v	m/s	Distance per time, direction *unspecified*
Velocity	$\boldsymbol{v}$	m/s	Distance per time, direction *specified*
Area	A	m^2	The product of two distances
Volume	V	m^3	The product of three distances
Density	ρ	kg/m^3	Mass per volume
Kinetic Energy	KE	J	Energy of motion. The heavier and faster something is, the more energy of motion it has. KE = ½ mv^2
Potential Energy	PE	J	Stored energy. The farther you pull something away from an equilibrium position, the more potential energy it has. Like lifting an object, or compressing a spring. PE = mgh (pulling against gravity)
Acceleration	a	(m/s) / s	How fast the speed or velocity is changing
Force	F	N	How hard something is pushing or pulling F = ma (Newton's second law)
Work	W	J	How much energy is spent or invested. W = Fd
Charge	q	coulombs	How many electrons or protons are collected in a certain location. One mole of electrons (6.022×10^{23}) has a charge of 96,500 coulombs (called Faraday's constant).

Questions to Ponder

a) *Of course, there are many more quantities that we observe and measure. Can you think of some?*
b) *Modern philosophers/scientists suggest that there is no such thing as 'time,' that it is merely derived from other quantities. How do we measure time?*
c) *How are PE and Work related?*
d) *How would you quantify paranormal or supernatural phenomena?*
e) *Einstein derived the formula $E = mc^2$. What are the implications? What do the units tell you about physical reality?*

Transducers

A transducer is a device that converts a quantity of interest into a conveniently measurable quantity. For example, a thermometer converts the temperature of a system into a distance. A ruler converts a distance into a number. A bathroom scale converts a weight (force due to gravity) into a rotation on a dial. Modern transducers typically produce a voltage (electrical potential energy) proportional to the quantity of interest (e.g. temperature, pressure, force, distance), so that a simple linear relationship can be used to obtain the quantity of interest (e.g. volts per degree).

Questions to Ponder

a) Our body contains many transducers. Each of our senses converts a physical phenomenon into a signal that the brain can interpret. How is this accomplished?
b) The speedometer in your car is a transducer. How does it work? There are both mechanical and electronic speedometers.
c) Can you think of other transducers in nature?
d) The concept of transducers can be extrapolated to many other areas of thought. For example, the worse the behavior, the higher the count of spankings. Try to generalize the idea of transducers. What other situations apply?

Accuracy vs. Precision, Analog vs. Digital

Physical quantities are *exactly* what they are (*profound, eh?*), and Nature is typically integerial or analog. Examples are: "How many electrons are there?" (integerial), or, "How high did you lift the object?" (analog). It is possible to *exactly* record the numbers of things ("I have three pennies in my pocket."), but when we measure analog quantities we must *approximate* them with numbers ("I weigh about 190.2 lbs."). The more decimal places we use to express an analog quantity, the more *precisely* we are approximating it. If the device we are using to measure a physical quantity is not calibrated correctly, then we may be *precisely* recording a result that is not *accurate*.

Once we have obtained a number which approximates a phenomenon, we need to record it. Traditionally, values are recorded in a laboratory notebook. In modern systems, electronic means (tapes, computers) are also used. This presents another issue; how do we represent a quantity in a computer? If the number is integerial and small enough, the number can be expressed *exactly* as an integer, routinely up to 2^{32} (4,294,967,296). Analog values are typically approximated to the degree of precision required by the situation, the data acquisition rate, and the amount of storage available. For example,

- In digital cameras there is a tradeoff between the quality of the photos and the number that can be stored. If you are planning to make large enlargements of the photos, you should use the highest resolution available to obtain the best looking print later. If you are merely documenting an event, then you can dramatically increase the number of photos you can store by storing the photos with less color and pixel resolution.

- If you are recording the music in a concert, then you typically only need to record it to 20 bits of resolution (one part in 2^{20} is about one part per million), since very few people can hear a difference that small. A recording of lower *fidelity* (fewer bits, lower precision) might still be sufficient if the range of the signal is small (not too loud or too soft).

Digital values are not only convenient for manipulation and storage, but also for transmission. For example, most radio signals are transmitted in analog form (AM and FM). Interference reduces the volume of the signal and/or produces static (noise produced when stray signals are amplified or the amplitude of the real signal is slightly changed). On the other hand, Digital signals are composed of "on/off" states (binary, bits), so transmission fidelity is *perfect* assuming all the bits are received. Let's say you are transmitting the number '153.' In binary, this number is represented (8 bits) as '10011001.' It doesn't matter how strongly the 'ones' are received, just so long as they are sufficiently strong that they can be distinguished from the background noise. In analog transmission, the number might be received as 154 or 151, reducing the fidelity. Digital cell phones, satellite TV, and the internet are examples of high-fidelity signal transmission via digital values.

Scientific Notation

Scientists routinely make and record a very large range of measurements. Of course, they are concerned about both accuracy and precision. Atomic clocks resolve atomic vibrations (0.000000000000001 seconds, or 10^{-15} seconds), and astronomers measure 'astronomical' distances and masses. The average distance from the Earth to the sun is 1.50×10^{11}m (150,000,000,000 meters) and the Earth's mass is 5.98×10^{24} kg (5,980,000,000,000,000,000,000,000 kilograms). Lots of zeroes, eh? Can you imagine having to write them out all the time? Accordingly, scientists have developed a convenient convention for precisely recording a large range of values, called *Scientific Notation*. (By the way, this is a great, intuitive opportunity to review logarithms.)

Scientific notation consists of three numbers (the *coefficient*, the *base*, and the *exponent*), but also includes some additional important ideas. Consider the mass of the earth, 5.98×10^{24}kg. If I were to express this quantity as 6×10^{24}kg I would not be incorrect, just less precise (one instead of three significant figures). Note that the number of digits in the coefficient reveals the precision (not the accuracy) to which the quantity is expressed. I am reminded of the Star Trek episode where Captain Kirk asks Spock how long it will take to get to their destination. Spock replies something like "22 hours, 3 minutes, and seven seconds." Captain Kirk smirks, and says that "about 22 hours" would have been a sufficient answer. In other words, the precision one uses to record and report data depends upon the precision of the original measurements, the subsequent calculations employed, and the intended use of the values.

Table 1. Sizing Things Up in Base Ten

ORDER	DECIMAL	PREFIX	EXAMPLE
10^{-15}	0.000000000000001	fempto-	atomic vibrations (fs), electron radius (fm)
10^{-12}	0.000000000001	pico-	wavelength of X-rays (pm)
10^{-10}	0.0000000001	(Angstroms, Å)	chemical bond lengths are in Å
10^{-9}	0.000000001	nano-	diameter of molecules (nm)
10^{-6}	0.000001	micro-	diameter of red blood cell (m)
10^{-3}	0.001	milli-	thickness of a dime (mm)
10^{-2}	0.01	centi-	Width of your pinky (2.54 cm/inch)
10^{-1}	0.1	deci-	Width of a hand (dm)
10^{0}	1	*n/a*	I'm about 2m tall
10^{1}	10	deca-	The crest of my roof is 1dam
10^{2}	100	hecto-	Close to the 100 yard dash
10^{3}	1,000	kilo-	A kilometer is about 0.62 miles
10^{6}	1,000,000	mega-	Millions. Lifetime earnings in dollars.
10^{9}	1,000,000,000	giga-	Billions. My computer has 2gbyte RAM
10^{12}	1,000,000,000,000	tera-	Trillions. Computer operations per second.
10^{15}	1,000,000,000,000,000	peta	How far light travels in a month (Pm)

Questions to Ponder

a) *Sometimes it is difficult for us to visualize very large and very small numbers. Try to think of something tangible in your experience for each order of magnitude.*
b) *How many molecules tall are in your body?*
c) *Light from the moon is about a second old. Light from the sun is about eight minutes old. In other words, when we 'look' at them, we are actually seeing what the moon was like one second ago, and what the sun was like eight minutes ago. When we see both at the same time, we are simultaneously observing two different moments in history.*

Significant Figures

The way one records and reports numbers is very important, especially in science. This is because the context and the number *itself* reveal how the measurements were made and the significance of the digits themselves. The convention in science is to indicate the measurement precision by the number of digits used, called *significant figures*. Zeroes acting as place-holders are not significant. Table 2 gives some examples:

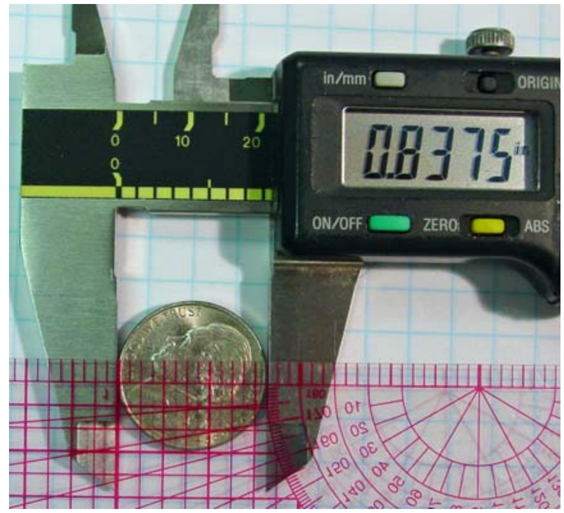
Table 2. Understanding Significant Figures

Number	Significant Figures	Scientific Notation	Explanation
1234	4	1.234×10^3	This number is assumed to be measured and reported to the nearest 1 (it could have been 1233 or 1235).
1230	3	1.23×10^3	Assumed to be ± 10, or between 1220 to 1240. (The zero is a place-holder.)
1030	3	1.03×10^3	Only the final zero is a place-holder
1200	2	1.2×10^3	Assumed to be ± 100, or between 1100 to 1300. (The zeroes are place-holders.)
1230 ± 1	4	1.230×10^3	The precision is unambiguous, and the uncertainty tells you that the last zero is significant.
73½ ± ½	3	$(7.35 \pm 0.05) \times 10^1$	Showing the ½ in scientific notation makes the precision unambiguous.
0.001	1	1×10^{-3}	The first two zeroes after the decimal point are place-holders, and the uncertainty in the number is assumed to be ±0.001
0.0010	2	1.0×10^{-3}	The first two zeroes after the decimal point are place-holders, and the uncertainty in the number is assumed to be ±0.0001

The context also matters. If someone asks me my height there are several ways I could answer, depending upon the context. At a party I might reply, "about six feet," and people know that I am not saying that it could be five feet or seven feet (6 ± 1 ft). They know I mean 'plus or minus an inch or so' (72″ ± 2″) because that is the convention in our culture. However, when I'm at the doctor's office, if the nurse asks me, I'll say 6ft 1½″, because I know they're trying to keep a precise medical record. Based upon my answer the nurse should record 73½″ on my chart. It would be a mistake to record 73.5″ or 73.50″, because the number 73.5 implies 73.5 ± 0.1 (three significant figures) and the number 73.50 implies 73.50 ± 0.01 (four significant figures), and I did not measure nor report my height to that degree of precision. Technically, the nurse should either re-measure me or ask me how precisely I was reporting the number. For example, I could say ‘to the nearest ¼ inch’ or ‘the nearest ½ inch.’

The issue of significant figures comes up all the time in making and reporting measurements, as the following examples illustrate, where I measure the dimensions of a nickel in British and metric units with both a ruler and a digital caliper.

Figure 1. Measurement in Inches



First, I measure the diameter of a nickel with both a ruler and a caliper. Note that the ruler has graduations every 1/16th of an inch, and that the caliper displays digits to the nearest 0.0005 inch *(you wouldn’t have known about the caliper resolution unless I’d told you)*. I’m pretty good with a ruler, and can reliably use it to within a ¼ of a sixteenth, or 1/64th. With the ruler I’d say it looks a little longer than 13/16ths, and so I’d record a measurement of 53/64 ± 1/64″. Measuring with the caliper, I’d report 0.8375 ± 0.0005″. Note that the caliper measurement is about thirty times more *precise* than the ruler, but they are equally *accurate*, since the range of ruler values includes the caliper value:

Device	***Minimumm Value***	***Maximum Value***	***Precision***
Ruler	53/64 - 1/64 = 0.813	53/64 + 1/64 = 0.844	(1/64)/(53/64) = 1.9%
Caliper	0.8375 – 0.0005 = 0.837	0.8375 + 0.0005 = 0.8380	0.0005/0.8375 = 0.06%

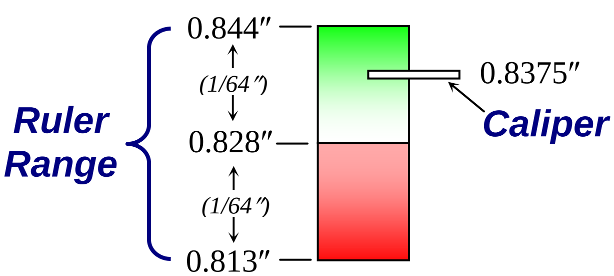
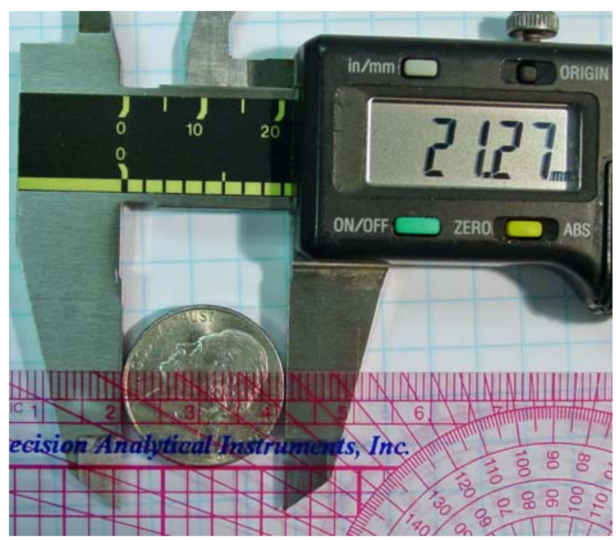


Figure 2. Measurement in Millimeters



Now, I push the “in/mm” button on my caliper and flip the ruler over to the mm scale (Figure 2). First, note that just because the units have changed, the precision of the device has not changed. In other words, 0.01/21.27 (0.05%) is about the same as 0.0005/0.8375 (0.06%). The manufacturer clearly understands significant figures! The value on the caliper is 21.27mm as expected, since there are 25.4mm per inch. In this case, I would record 21.27 ± 0.01mm in my notebook.

Using the ruler, I measure less than 22mm, and since I am only good to about 1/3 mm with a ruler I would record 21 2/3 ± 1/3 mm in my notebook. Note that in this case I am less *accurate* with the ruler, as my range of ruler values does not include the correct value. Perhaps I am not as good with a ruler as I thought!